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LARGE-ARRAY SIGNAL AND NOISE ANALYSIS

Quarterly Report No. 6

1 October 1967 through 31 December 1967

Frank H. Binder, Program Manager

TEXAS INSTRUMENTS INCORPORATED
Science Services Division
P. O. Box 5621
Dallas, Texas 75222

Contract No. AF 33(657)-16678

Prepared for

AIR FORCE TECHNICAL APPLICATIONS CENTER
Washington, D. C. 20333

Sponsored by

ADVANCED RESEARCH PROJECTS AGENCY
ARPA Order No. 599
AFTAC Project No. VT/6707

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TEXAS INSTRUMENTS
INCORPORATED
SCIENCE SERVICES DIVISION

11 January 1968

Air Force Technical Applications Center
VELA Seismological Center
Headquarters, USAF
Washington, D C. 20333

Attention: Captain Carroll F. Lam

Subject: Sixth Quarterly Report Covering Period October 1, 1967
          Through December 31, 1967.

AFTAC Project No.:	VT/6707
Project Title:	Large Array Signal and Noise Analysis
ARPA Order No.:	599
Name of Contractor:	Texas Instruments Incorporated
Date of Contract:	16 May 1966
Amount of Contract:	$1,083,696
Contract Number:	AF 33(657)-16678
Contract Expiration Date:	25 June 1968
Program Manager:	Frank H. Binder
	Area Code 214
	238-3473

Gentlemen:

Approval of the one year extension in the amount of $490,023 was concluded during the last quarter.

Below is set forth the work progress against the major tasks remaining under the contract.

PUBLICATION OF SPECIAL REPORTS DEALING WITH WORK PREVIOUSLY COMPLETED

The following special reports dealing with previously completed work were published during the past quarter.

LASA Special Report No. 3 - SUBARRAY PROCESSING.

LASA Special Report No. 6 - ANALYSIS OF SUBARRAY WAVENUMBER SPECTRA.

LASA Special Report No. 12- ANALYSIS OF LONG PERIOD NOISE.

LASA Special Report No. 13-NOISE COHERENCE AMONG SUBARRAYS.

LASA Special Report No. 14-WIENER NON-TIME STATIONARY PROCESSING.

The following reports are in different stages of publication and should be completed by February, 1968.

LASA Special Report No. 4 - SPACE AND TIME VARIABILITY OF THE LASA NOISE FIELD.

LASA Special Report No. 10 -EQUALIZATION STUDIES.

LASA Special Report No. 11-RESOLUTION OF EVENTS.

SHORT PERIOD NOISE ANALYSIS

K-line spectra have been run on two summer noise samples. The spectra have not yet been analyzed. These spectra should give an indication of any fundamental seasonal changes.

A report on the summer noise analysis will be published during the coming quarter. This report will describe the wavenumber spectra of the summer noise samples and compare them with the previously published results for the winter noise samples. (LASA Special Scientific Report No. 6, ANALYSIS OF SUBARRAY WAVENUMBER SPECTRA.)

A study of locally generated noise at the LASA will be begun in February. The purpose of this study will be to try to identify and describe noise sources within the large array.

LONG PERIOD NOISE STUDIES

- **Analysis of Summer Noise Samples**

The purpose of this study is to compare the characteristics of the long-period noise field during the summer months with those observed during winter months (Special Report No. 12). Five noise samples will be analyzed; these have been demulitplexed, transferred to the System 360, and are presently being despiked and plotted. Analysis techniques will parallel those described in Special Report No. 12.

- **Further Study of the Noise Below 0.05 cps**

The long noise sample to be used to study the noise below 0.05 cps consists of five back-to-back 80-minute tapes. These tapes have been de-multiplexed, transferred to the System 360, and are presently being despiked and plotted. Plots have shown that the first tape has a small surface wave arrival (primarily on the verticals), and the fifth tape has an 89-point segment of bad data. The feasibility of including part or all of these tapes in the long noise sample is presently being investigated. This long noise sample will allow more careful estimates of the low frequency spectra and coherence.

In addition, a program is being written to demultiplex the microbarographic data associated with the long noise sample. It is anticipated that preliminary data handling (including the microbarographic information) will be completed early in February, 1968.

The microbarograph data will be used to check for correlation between pressure fluctuations and long period seismometer output at low frequencies ($f < .05$ cps).

- **Continued Studies of Love-Wave Noise Energy**

    Past studies (LASA Special Scientific Report
    No. 12) have indicated that point-like sources
    of Rayleigh mode energy may also be sources
    of Love mode energy.

    Some horizontal components have been rotated
    in-line and transverse to the very strong storm
    source indicated in the February 7 noise sample.
    Power spectra and coherences have been generat-
    ed to see if there are indications that this storm
    source is generating Love mode energy.

    In the region near the 7-sec microseism peak,
    the horizontal components are generally 5-10 db
    noisier than the vertical components (LASA No.
    12). This may be the result of (1) Love mode
    energy, (2) non-seismic noise, (3) change in
    excitation of horizontal to vertical motion of
    Rayleigh mode energy.

    Possibility No. 2 is unlikely based on the
    apparent coherence of the horizontal components
    in this frequency range (LASA No. 12). Possi-
    bility No. 3 can be fairly well confirmed or
    denied by calculating the excitation functions for
    the LASA crust. These calculations are being
    made and will be completed and reported during
    the coming quarter.

- **Study of Vertical Component Noise Not Predictable
  From Horizontal Components**

    In general about 90% of the noise power on the
    verticals is predictable from nearby horizontals
    at the 15-18-sec microseism peak. In order
    to try to dissect the remainder of the vertical
    component power, a simultaneous prediction
    of all verticals from all horizontals will be done.

    This operation is conveniently accomplished in
    the frequency domain from the crosspower matrix

of all channels (vertical and horizontal).
This operation yields a conditional covariance
only; that is, the covariance matrix of the
unpredictable (from horizontals) part of the
vertical component noise. The resulting
covariance matrix can be used for coherency
and wavenumber analysis.

Using such a large number of channels
(potentially 21V, 21E and 21N) requires a very
long noise sample to obtain adequate statistics.
The long noise sample described above will be
used for this study. Depending on how long a
sample is finally usable, the number of channels
will be adjusted to give adequate statistics.

- Stability of Long Period Noise

Time domain signal extraction and prediction
filters were designed on one noise sample and
applied to the design interval and to another time
gate separated by about 6 hours.

This data is being combined with a study of the
usefulness of horizontals in P-wave extraction
which is not yet complete. A report on this
study will be forthcoming during February.

- Correlation of Storms at Sea with the Long
Period Noise Sources

A study of the correlation between the long
period noise sources (as seen in wavenumber
spectra) and sea conditions has been completed.
The correlation appears to be meaningful but
many details are perplexing. The results are
detailed in LASA Special Scientific Report No.
17 which has been submitted to AFTAC for
approval.

LONG PERIOD SIGNAL ANALYSIS AND DISSECTION

A. EXTRACTION OF SIGNALS FROM AMBIENT NOISE

The goal of this work is to evaluate various techniques for extracting the bodywave phases and surface wave modes of an event in the presence of ambient noise. Both off- and on-line procedures will be studied. In the off-line case we assume that the approximate epicenter and P-wave arrival time are known. For this case, a number of specific questions arise.

1) What is the beam steer performance?

2) Can MCF provide superior performance or equivalent performance with reduced array size and number of elements?

3) How does microseismic storm noise affect our results?

4) How useful are the horizontals?

To gain some insight into the problem, 70-minute noise samples from December 2 and 3, 1966 have been analyzed. First the following averages (straight sums) of vertical traces were computed.

$$
\begin{aligned}
&\text{9 channels - A0, C ring and D ring}\\
&\text{12 channels - A0, C ring, D ring, and E ring except E1}\\
&\text{(Dec. 2) 18 channels - all except E1, F1, and F3}\\
&\text{(Dec. 3) 20 channels - all except F1}
\end{aligned}
$$

Next, infinite velocity signal extraction filters were designed for the first two combinations listed above. The December 3 noise sample was used to develop noise statistics for the filter design, and the filters were applied to both noise samples. Ratios of the A0 noise spectrum to the spectra of the outputs of the various processors were computed. The results will be published in a special report. The following points of interest are noted.

1) Noise suppression by the beam steer processors, while differing in small details, is essentially the same for these two noise samples.

2) In the range 0.05 to 0.20 Hz for the December 3 sample, each MCF is everywhere superior to the corresponding beam steer. Either MCF is superior to any of the beam steer processors except for a small range near 0.14 Hz where the 20-channel

average outperforms them.

3) Performance of the MCF processors on the December 2 sample is about 2 - 3 db poorer than on the December 3 sample.

In addition, a prediction error MCF was designed using the A0, B1, B3, and B4 horizontals to predict the A0 vertical. This processor was notably inferior to any of the others in suppressing the noise. The reasons for this are not fully understood.

Finally an infinite velocity signal extraction MCF was designed using all the A0 and B ring sensors except the B1 vertical and B2 horizontals. The result was just slightly inferior to the 9 and 12 channel MCF systems. Since the B ring horizontals were not effective in predicting the A0 vertical this suggests using the A0 and B ring verticals. This is in progress but has not been completed.

These results indicate that multi-channel filtering of the inner rings may be as effective as beam steering the entire array for bodywave signal extraction. This will be further investigated by actually performing the extraction on recorded signals. In designing the MCF processors there is some question regarding the best way to generate the noise statistics. Ordinarily, it would seem best to use a noise sample immediately preceding the event. If, however, there is overlap between various phases of the event, the use of a time gate including the entire signal might result in better extraction of the individual phases. This problem will be studied.

The type of analysis performed on the 2 December noise samples will be repeated as an initial approach to the problem of surface wave extraction. Here rather than straight sum we will use time shift and sum, and the MCF processors will be designed for a surface wave signal model. In this way we hope to get an indication of which types of processing are apt to be effective on this problem. The December noise samples used in the above studies were dominated by point-like noise sources indicating that they contained a substantial amount of microseismic storm noise. The presence of such noise is probably more significant in the surface wave extraction problem than for bodywave extraction. Therefore, this analysis will include noise samples representing the quiet ambient noise case and the storm noise case.

B. EXTRACTION OF SIGNAL FROM STRONG INTERFERING EVENT

This effort deals with the extraction of event phases which arrive simultaneously with a strong signal from another event. The problem is to suppress the interfering signal while retaining and dissecting the target event. Several off-line approaches to the problem will be studied.

1) Beam steer. This technique is straightforward. In the case of the surface wave phases of the target event, consideration of dispersion in doing the beam steer should provide some improvement over simple time shift and sum. This effect, however, is expected to be small.

2) Another method is to time shift all n traces to make the interfering signal align (again possibly taking dispersion into account). Then by subtracting one trace from all the rest one obtains n-1 traces containing distorted target signal and ambient noise. The signal is reconstructed and ambient noise reduced by subsequent MCF.

   If the signal and the interfering signal exactly fit a plane wave hypothesis one could entirely eliminate the interfering signal while passing the desired signal. Obtaining very large rejection probably depends on forcing the best possible fit to a plane wave model. Interpolation for time shifts of less than one second and perhaps frequency dependent equalization will be attempted.

3) The third method is to design a MCF to operate directly on the seismic traces. A problem here is to determine the best way to generate noise statistics for the filter design. One way is to use a theoretical noise model. If one uses measured noise the question is whether to use a time gate containing all of the interfering event or just that portion which precedes the arrival of the target event.

4) A fourth technique which may be useful is to form one beam toward the interfering signal. One then uses the latter to predict the former and calls the prediction error the signal estimate. A technique of this type would probably be easy to implement on-line.

In view of the nature of the principal noise component in this problem, it appears likely that results superior to those realized by a beam steer processor should be available. In order to test this hypothesis it is necessary

to first do the beam steer and then to perform other types of processing. To accomplish this the Rayleigh phase from a California event has been imbedded in the Rayleigh phase from a Greenland Sea event with its peak amplitude scaled to half that of the Greenland Sea event. Epicentral directions were 225° and 20° respectively. A simple time shift and sum aimed at the California event resulted in about a 4:1 improvement in the amplitude of that event over the interfering one. This work will be continued by evaluating more complex approaches to the problem. Similar analyses will be performed by compositing events with lesser amounts of azimuthal separation.

C. OTHER STUDIES

The lateral refraction of surface waves is a matter of consequence in this work. This results in propagation vectors at the array which differ from the great circle path from the epicenter to the array. If effective enhancement of the signal is to be accomplished the propagation vectors for the signal and any interfering events must be determined. A technique employing the outputs of three seismometers to estimate the speed and direction as a function of frequency has been developed. This has been applied to the California and Greenland Sea events. Analysis of other events in our library will be performed.

The study of high resolution wavenumber spectra for the separation of event phases has been continued. As reported previously this technique has been applied to an event which has clear, time-separated compressional, shear, and Rayleigh phases. When the time gate used for computing the spectra included the Rayleigh phase, the spectra did not show the shear energy. Subsequently this has been repeated with a data gate containing only the shear phase. In this case, the shear energy was well defined in the resulting f-$\vec{k}$ plots. Continued work in this area is planned.

HIGH RESOLUTION WAVENUMBER SPECTRA RESEARCH

Probabilistic processing in the frequency-wavenumber domain is developed. Conventional high resolution frequency-wavenumber spectra are shown to be equivalent to a probabilistic processor when the noise field is assumed to be Gaussian and uncorrelated between sensors. Coherent noise fields are shown to require more complex processors for best results.

A. DERIVATION OF THE PROBABILISTIC PROCESSING EQUATION

Frequency domain probabilistic processing is a method using seismic array data to detect plane wave signals in the presence of ambient seismic noise. This method is based on the assumption that the array data is Gaussian with mean zero and known crosspower matrix $[Q_N]$ or $[Q_{s+n}^m]$,

depending on the absence or presence of signal. The array output is considered to be K complex numbers resulting from the collateral Fourier transform of time gates from K channels of simultaneous array time domain data.

The criteria for estimating the presence or the absence of a plane wave signal is analogous to testing the simple hypothesis that the observed data has a crosspower matrix $[Q_N]$ with the alternate multiple hypotheses that the data is from a set of crosspower matrices of the form $[Q_{S+N}^m]$. The superscript m refers to a particular plane wave signal model.

The derivation of the Gaussian multichannel sampled data processing equation starts with the multi-variate Gaussian probability distribution. [1]

$$P(x_1, x_2, \ldots, x_K) = \frac{1}{(2\pi)^{K/2} |X|^{1/2}} \exp\left\{-\tfrac{1}{2} \{x\}_i^{*T} [X]^{-1} \{x\}_i\right\}.$$ (II-1)

In this equation, matrix notation is used as follows:

$\{X\}_i$ or X_i $= i^{\text{th}}$ set of complex data values $x_1, x_2, \ldots, x_K$ with x_l being the Fourier transform of the lth time gate.

$[X]$ or X $=$ crosspower matrix of the data, i.e., $X_{ij} = \overline{X_i^* X_j}$.

The superscript T designates the matrix transpose; the superscript -1 designates the inverse matrix; and the superscript $*$ signifies the matrix complex conjugate. Vertical lines bordering a square matrix indicates the matrix determinant.

In Equation II-1, K is the number of data values. In a multichannel problem, K would usually be equal to the number of channels.

The detection of a plane wave signal in the presence of ambient seismic noise can be formulated as a multi-possibility multichannel Gaussian problem. That is, one can assume that the data, X_i, is a sample of a stationary Gaussian time series whose K by K crosspower matrix is noise, $[Q_N]$, or one of a set of signal plus noise matrices, $[Q_{S+N}^m]$.

To completely specify the problem using a Bayesian approach, the a priori probabilities $P(Q_N)$, $P(Q_{S+N}^m)$, of the possibilities must be known or estimated. The problem is then to obtain the a posteriori probabilities of noise alone, $P(Q_N \mid X_i)$, and of signal plus noise, $P(Q_{S+N}^m \mid X_i)$, by analyzing the data X_i. It is understood that $[Q_{S+N}^m]$ is to range over all allowable plane wave signal models.

With regard to the seismic detection problem, Q_N stands for ambient seismic noise and $P(Q_N)$ would be the fraction of time that no signal was present. Q_{S+N}^m would stand for a particular signal model in the presence of noise and $P(Q_{S+N}^m)$ would be the fraction of time that this signal is present.

A statement of the a posteriori probability using Bayes theorem for noise alone is

$$P(Q_N \mid X_i) = \frac{P(X_i \mid Q_N)}{P(X_i)} \, P(Q_N) \tag{II-2}$$

and for signal plus noise the a posteriori probability is

$$P(Q_{S+N}^m \mid X_i) = \frac{P(X_i \mid Q_{S+N}^m)}{P(X_i)} \, P(Q_{S+N}^m) \, . \tag{II-3}$$

From Equation II-1, one has

$$P(X_i \mid Q_N) = \frac{1}{(2\pi)^{K/2} \mid Q_N \mid^{1/2}} \exp \left\{ -\tfrac{1}{2} X_i^{*T} [Q_N]^{-1} X_i \right\} \tag{II-4}$$

and

$$P(X_i \mid Q_{S+N}^m) = \frac{1}{(2\pi)^{K/2} \mid Q_{S+N}^m \mid^{1/2}} \exp \left\{ -\tfrac{1}{2} X_i^{*T} [Q_{S+N}^m]^{-1} X_i \right\} \tag{II-5}$$

with

$$P(X_i) = \frac{P(Q_N)}{(2\pi)^{K/2}|Q_N|^{1/2}} \exp\left\{-\tfrac{1}{2} X_i^{*T}[Q_N]^{-1} X_i\right\}$$

$$+ \sum_m \frac{P(Q_{s+N}^m)}{(2\pi)^{K/2}|Q_{s+N}^m|^{1/2}} \exp\left\{-\tfrac{1}{2} X_i^{*T}[Q_{s+N}^m]^{-1} X_i\right\} . \tag{II-6}$$

From this, we have the a posteriori probabilities,

$$P(Q_N|X_i) = \frac{\frac{P(Q_N)}{|Q_N|^{1/2}} \exp\left\{-\tfrac{1}{2} X_i^{*T}[Q_N]^{-1} X_i\right\}}{(2\pi)^{K/2} P(X_i)} \tag{II-7}$$

and

$$P(Q_{s+N}^m|X_i) = \frac{\frac{P(Q_{s+N}^m)}{|Q_{s+N}^m|^{1/2}} \exp\left\{-\tfrac{1}{2} X_i^{*T}[Q_{s+N}^m]^{-1} X_i\right\}}{(2\pi)^{K/2} P(X_i)} . \tag{II-8}$$

The denominator of Equations II-7 and II-8, which is proportional to $P(X_i)$, normalizes $P(Q_N|X_i)$ and the $P(Q_{s+N}^m|X_i)$ so that their sum is one. For Gaussian distribution, the Bayes estimates of II-7 and II-8 are optimum processes since the overall probability of error is minimized.[2]

Equations II-7 and II-8 are the basic equations for their multi-possibility probabilistic processor using Gaussian distributions. There are, however, two other essentially equivalent forms of Equations II-7 and II-8. One is the probability ratio form,

$$\frac{P(Q_{s+N}^m|X_i)}{P(Q_N|X_i)} = \frac{|Q_N|^{1/2}}{|Q_{s+N}^m|^{1/2}} \cdot \frac{P(Q_{s+N}^m)}{P(Q_N)} \exp\left\{-\tfrac{1}{2} X_i^{*T}[Q_{s+N}^{-1} - Q_N^{-1}] X_i\right\} . \tag{II-9}$$

Equation II-9 gives the ratio of the a posteriori probabilities of $P(Q_{s+N}^m)$ and $P(Q_N)$. This form avoids computing the normalization factor, $P(X_i)$, used in Equations II-7 and II-8. However, this normalization factor can be found from the probability ratios since the sum of the probabilities is one.

The second equivalent form is the log ratio,

$$\log_e \frac{P(Q_{s+n}^m | X_i)}{P(Q_n | X_i)} = \log_e \frac{|Q_n|^{1/2} P(Q_{s+n}^m)}{|Q_{s+n}^m|^{1/2} P(Q_n)} - \frac{1}{2} X_i^{*T} [Q_{s+n}^{m^{-1}} - Q_n^{-1}] X_i \quad . \tag{II-10}$$

This form shows clearly that if

$$X_i^{*T} [Q_{s+n}^{m^{-1}} - Q_n^{-1}] X_i > 2 \log_e \frac{|Q_n|^{1/2} P(Q_{s+n}^m)}{|Q_{s+n}^m|^{1/2} P(Q_n)}$$

then

$$P(Q_n | X_i) > P(Q_{s+n}^m | X_i)$$

$$\tag{II-11}$$

i.e., the data is more likely noise.

Thus the problem is to determine which is more likely, Q_n or Q_{s+n}. This is done by calculating the test statistic,

$$X_i^{*T} [Q_{s+n}^{m^{-1}} - Q_n^{-1}] X_i \tag{II-12}$$

from the data, X_i, for each plane wave signal model $[Q_{s+n}^m]$. The number obtained is compared with the threshold value

$$C_m = 2 \log_e \frac{|Q_n|^{1/2} P(Q_{s+n}^m)}{|Q_{s+n}^m|^{1/2} P(Q_n)} \tag{II-13}$$

which is a scalar constant for a particular data set when all signal models are assumed to be equally likely. Since the plane wave signal models have cross-power matrices, $[Q_{s+n}^m]$, which are functions of the vector wavenumber, the test statistic of Equation II-10 can be contoured as a function of wavenumber. In the resulting contoured plot, the smaller numbers indicate the greater probability that a plane wave signal is present.

The use of the likelihood ratio test statistic of Equation II-11 is favored over direct use of the a posteriori probabilities of Equation II-7 and II-8 because of the difficulty in establishing the a priori probabilities $P(Q_{S+N}^m)$ and $P(Q_N)$. Equation II-11 has a threshold term containing the a priori probabilities but the size of the threshold is not of paramount importance. The relative magnitude of Term II-12 for different data sets, X_i, or different signals in the same data set, contains significant information as to probable event location.

B. PROBABILISTIC PROCESSING: LARGE ARRAY

For convenience, the likelihood ratio test statistic is duplicated below,

$$X_i^{*T} [Q_{S+N}^{m} - Q_N^{..}] X_i > 2 \log_e \frac{|Q_N|^{1/2} P(Q_{S+N}^{m})}{|Q_{S+N}^{m}|^{1/2} P(Q_N)} \qquad \text{(III-1)}$$

and will be referred to as the probabilistic processor.

1. Noise Crosspower Matrix Estimation-Uncorrelated Gaussian Noise Field

The implementation of Equation III-1 becomes simplified when the ambient seismic noise is assumed to be uncorrelated between channels and to be Gaussian. For a large array such as LASA, this noise field assumption has been experimentally shown to be valid.[3]

For the above mentioned noise field, the crosspower matrices $[Q_{S+N}^m]$ and $[Q_N]$ are simplified to

$$[Q_{S+N}^m] = [VV^{*T} + w I] \qquad \text{(III-2)}$$

and

$$[Q_N] = [w I] \qquad \text{(III-3)}$$

where

$$V = \left\{ \begin{array}{c} e^{j\,\vec{k}\cdot\vec{\pi}_1} \\ e^{j\,\vec{k}\cdot\vec{\pi}_i} \\ \vdots \\ e^{j\,\vec{k}\cdot\vec{\pi}_k} \end{array} \right\}$$

With

$\vec{k}$ = vector wave number

$\vec{\pi}_p$ = p^{th} array coordinate corresponding to the pth data channel

and

I = identity matrix

w = scalar multiplier equal to the magnitude of the power of the uncorrelated noise

The superscript, m , has been dropped from the signal plus noise crosspower matrix since the plane wave signal will hereafter be implicitly defined as a function of vector wavenumber, $\vec{k}$.

The inverse crosspower matrices become

$$[Q_{S+N}]^{-1} = \frac{1}{w}\left[I - \frac{1}{(w + V^{*T}V)}\,V V^{*T}\right]^{\dagger}$$

(III-4)

and

$$[Q_N]^{-1} = \frac{1}{w}[I] \quad .$$

(III-5)

Substitution of Equations III-4 and III-5 into the left side of Equation III-1 results in the probabilistic processor assuming the form

---

$\dagger$ The formula used is

$$(A + XY^{*T})^{-1} = R - R X\,(1 + X^{*T}R Y\,)^{-1}Y^{*T}R$$

where X and Y are row matrices (vectors) and $R = A^{-1}$. Both A and $(A + XY^{*T})$ are assumed to be nonsingular.

$$X_i^{*T}[Q_{s+N}^{-1} - Q_N^{-1}]X_i = -\frac{1}{w(w+V^{*T}V)}(V^T X_i^*)(V^T X_i^*)^* >$$

$$2 \log_e \frac{|Q_N|^{1/2} P(Q_{s+N})}{|Q_{s+N}|^{1/2} P(Q_N)} \qquad . \qquad \text{(III-6)}$$

After some algebraic manipulation, Equation III-6 becomes

$$2 \log_e \frac{|Q_N|^{1/2} P(Q_{s+N})}{|Q_{s+N}|^{1/2} P(Q_N)} - \frac{1}{w(w+V^{*T}V)}(V^T X_i^*)(V^T X_i^*)^* > 0 \qquad . \qquad \text{(III-7)}$$

Equation II-11, of the previous section, states that if this inequality is true, then more probably only noise is present.

The calculation on the left side of inequality is conveniently displayed as a function of discrete vector wavenumbers in a format similar to that of a frequency wavenumber spectra. A plane wave signal would appear as a low magnitude area in the wavenumber plot.

2. Equivalence to High Resolution Frequency-Wavenumber Spectra and Time Domain Beam Steer

High resolution frequency-wavenumber spectra are computed from a frequency domain multichannel filter. In the filter synthesis, the signal crosspower matrix is assumed to be uncorrelated Gaussian noise and the noise crosspower matrix is measured data.[4]

Multichannel filter design at a single frequency is

$$[w I + X_i X_i^{*T}] F_n^* = \Gamma_n \qquad \text{(III-8)}$$

with

$$F_n^{\nu} = \begin{Bmatrix} f_1 \\ f_2 \\ \vdots \\ \vdots \\ f_K \end{Bmatrix} \quad , \quad \Gamma_n = \begin{Bmatrix} 0 \\ \vdots \\ \omega \\ \vdots \\ 0 \end{Bmatrix} = n^{th} \text{ channel is reference}$$

where K, ω, X_i and I have the definitions given in previous sections. The subscript, n, ($1 \leq n \leq K$) specifies the reference channel used in the filter design.

The solution to Equation III-8 using the nth channel as reference is

$$F_n^* = \frac{1}{\omega} \left[I - \frac{1}{(\omega + X_i^{\nu T} X_i)} X_i X_i^{\nu T} \right] \Gamma_n \qquad . \tag{III-9}$$

A high resolution frequency-wavenumber spectra may be computed from Equation III-9 with a single sensor as reference, or

$$V^T F_n^* F_n^T V^{\nu} \tag{III-10}$$

V remains unchanged from the earlier definition.

Summation of the responses from using each channel as reference results in

$$\sum_n V^T F_n^{\nu} F_n^T V^{\nu} = V^T \left[\sum_n F_n^{\nu} F_n^T \right] V^{\nu} \tag{III-11}$$

Let $\beta_n = \frac{1}{\omega} \Gamma_n$.

Then Equation III-12 becomes

$$\sum_n V^T F_n^* F_n^T V^* = V^T \left[\sum_n \beta_n^* \beta_n^T \right] V^*$$

$$- \frac{2}{\omega + X_i^{*T} X_i} V^T \left[\sum_n \beta_n^* \beta_n^T \right] X_i X_i^{*T} V^*$$

$$+ \frac{1}{(\omega + X_i^{*T} X_i)^2} V^T X_i X_i^{*T} \left[\sum_n \beta_n^* \beta_n^T \right] X_i X_i^T V^*.$$

(III-12)

However, when all sensors are used as reference

$$\sum_n \beta_n^* \beta_n^T = I$$

(III-13)

and Equation III-12 simplifies to

$$V^T V^* - (V^T X_i^*)(V^T X_i^*)^* \frac{(2\omega + X_i^{*T} X_i)}{(\omega + X_i^{*T} X_i)^2} .$$

(III-14)

      Comparison of Terms III-7 and III-14 reveals that the probabilistic processing equation and the high resolution frequency-wavenumber spectra (when every sensor is used as reference) compute a test statistic of the same form. That is, both test statistics consist of a constant minus an amplitude scaled frequency domain beam steer.

Since the presently used time domain beam steers are
steered for a fixed velocity, their outputs would be essentially equivalent
to summing over like velocities the probability pictures (high resolution
spectra or probalistic processing plot) over all frequencies in the bandwidth
of the time domain processor.

As shown above for a random noise field, probalistic processing
is equivalent to beam steering or the high resolution spectra using all sensors
as reference.

C. PROBABILISTIC PROCESSING: SMALL ARRAY

As mentioned in an earlier section, the seismic noise field
may be assumed to be Gaussian and uncorrelated between sensors for large
arrays such as LASA. However, as the array aperture is decreased, the noise
field between sensors becomes correlated. The simple processor described in
Section III is no longer optimum.

One would expect signal detection capability to improve for small
arrays if a better estimate of the noise crosspower matrix were available.
Since the noise field is non-time-stationary, the estimate should vary as a
function of the prevailing noise conditions.

1. Noise Crosspower Matrix Estimation - Measured Data

At a single frequency, a time varying noise field can be
characterized by a crosspower matrix computed using an exponentially weighted
recursive least squares scheme.[5] Noise crosspower matrix updating assumes
the form

$$[Q_N]_{i+1}^{-1} = \exp\left(\frac{\Delta t}{\tau}\right)\left\{[Q_N]_i^{-1}\right.$$

$$\left. - [Q_N]_i^{-1} X_{i+1}\left[\exp\left(\frac{-\Delta t}{\tau}\right) + X_i^{*T}[Q_N]_i^{-1} X_{i+1}\right]^{-1} X_{i+1}^{*T}[Q_N]_i^{-1}\right\} \qquad (IV-1)$$

where Δt = time interval between data samples
τ = time constant of data decay
i = interation index referring to a particular time

The rate at which past estimates of the noise crosspower matrix are decayed is a function of γ. A rapidly varying noise field would necessarily require a more rapid taper for successful tracking.

Since noise field measurements in a signal interval are biased by the presence of the signal, these measurements should not be included in the noise crosspower matrix estimation of Equation IV-1. A decision criteria, for including or not including measurements, based on amplitude scan of the probabilistic plot could be used. When a measurement is not used in the crosspower matrix estimation, the matrix $[Q_{s+N}]_{i+1}$ will remain unchanged from $[Q_{s+N}]_i$.

2. Adaptive Probabilistic Processor

Using the noise field estimate of Equation IV-1, the signal plus noise crosspower matrix with a plane wave signal model becomes

$$[Q_{s+N}]_{i+1}^{-1} = \left[VV^{*T} + [Q_N]_{i+1} \right]^{-1} \tag{IV-2}$$

$$= [Q_N]_{i+1}^{-1} - [Q_N]_{i+1}^{-1} V \left(1 + V^{*T}[Q_N]_{i+1}^{-1} V \right)^{-1} V^{*T}[Q_N]_{i+1}^{-1} .$$
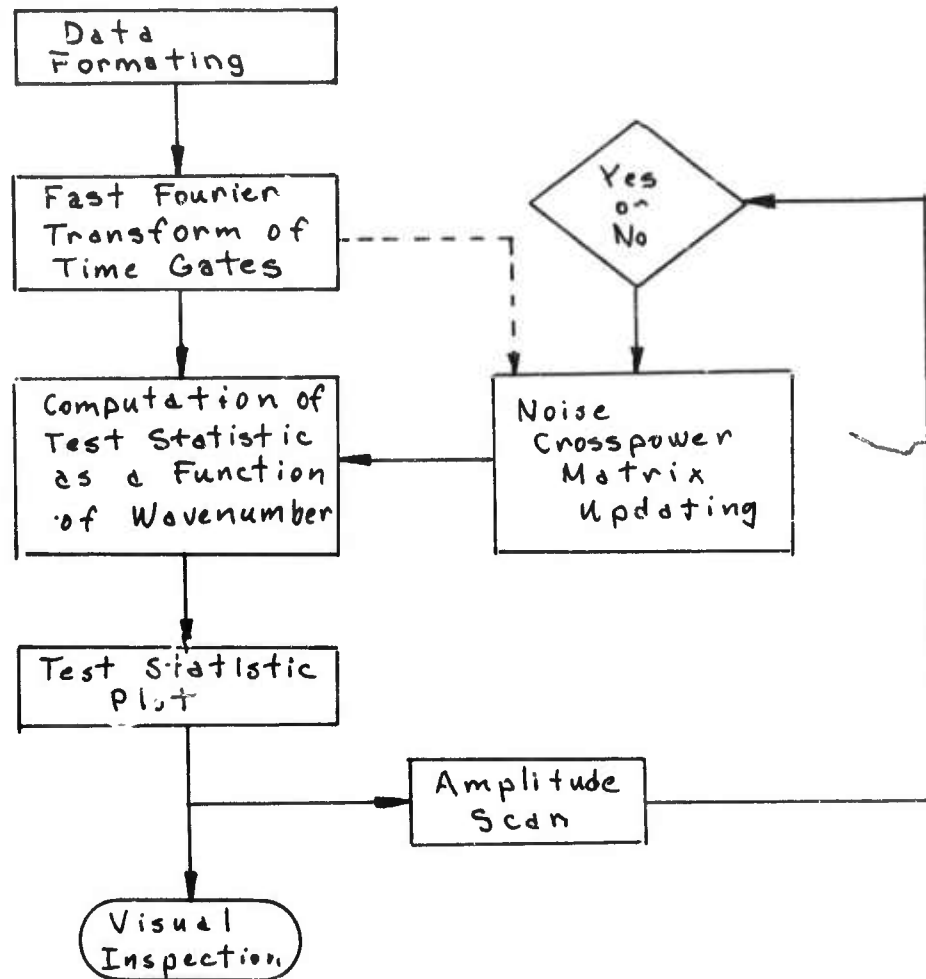
Substitution of Equations IV-2 and IV-1 into Equation III-1 produces an adaptive probabilistic processor which is optimum when the noise field is non-time-stationary and correlated between channels. The processor test statistic is

$$2 \log_e \frac{|Q_N|^{1/2} P(Q_{s+N})}{|Q_{s+N}|^{1/2} P(Q_N)} \tag{IV-3}$$

$$- \frac{1}{1 + V^{*T}[Q_N]_{i+1}^{-1} V} \left\{ V^{*T}[Q_N]_{i+1}^{-1} X_{i+1} \right\}^T \left\{ X_{i+1}^T [Q_N]_{i+1}^{-1} V \right\} > 0 .$$

As is evident, the inclusion of a non-random noise field complicates the computation of the test statistic.

D. PROBABILISTIC PROCESSOR PROGRAM IMPLEMENTATION

Wavenumber displays of the test statistic produced by a probalistic processor can be implemented through a series of programs represented by the flow chart shown below.

Individual programs of this series can be combined into a single program which will take multichannel time series data as input and sequentially output wavenumber likelihood ratio plots.

E. STUDY OF LOCATION SCHEME STATISTICS

A comparison of the various frequency-wavenumber $(f\text{-}\vec{k})$ location schemes in several signal and noise environments is being conducted. The objective of this study is to determine for each scheme the probability of correctly locating plane wave events in both correlated and uncorrelated noise fields at varied signal-to-noise ratios. With this information, it will be possible to choose the best scheme for locating teleseismic events in wavenumber space.

The four location schemes being compared are

- Convention wavenumber spectra

$$V^* X X^* V$$

- Probabilistic processing for single plane wave signal

$$X^*[N + VV^*]^{-1}X = X^*N^{-1}X - \frac{X^*N^{-1}VV^*N^{-1}X}{(1 + V^*N^{-1}V)}$$

- Two forms of high resolution wavenumber spectra

HR 1

$$V^*[N + XX^*]^{-1}V = V^*N^{-1}V - \frac{V^*N^{-1}XX^*N^{-1}V}{(1 + X^*N^{-1}X)}$$

and HR 2

$$V^*[N + XX^*]^{-2}V = V^*N^{-2}V - \frac{(2 + 2X^*N^{-1}X - X^*N^{-\frac{1}{2}}X)}{(1 + X^*N^{-1}X)^2} V^*N^{-1}XX^*N^{-1}V$$

N is noise covariance matrix
X is vector of data transforms
V is unit plane wave transform for various signal models

Initially an array geometry equivalent to the subarray locations (center seismometer) on the A, B, C, and D rings of the LASA was used and the signal transforms were obtained from this geometry. The noise covariance matrix N becomes a constant time the identity matrix ρI since the noise is largely incoherent for arrays of this size.[6] In this situation the above schemes reduce to

• Conventional wavenumber spectra

$$V^*XX^*V$$

• Probabilistic processing

$$X^*X - \frac{X^*VV^*X}{(1 + V^*V)}$$

• HR 1

$$V^*V - \frac{V^*XX^*V}{(1 + X^*X)}$$

• HR 2

$$V^*V - \left[\frac{2 + X^*X}{(1 + X^*X)^2}\right] V^*XX^*V$$

The term X^*X is the sum of the power in each channel at the frequency being processed and V^*V is a scalar equal to the number of channels. Thus the last three techniques are different combinations of the conventional wavenumber spectra, the sum of the power in each channel, and the number of channels.

In both correlated and uncorrelated noise fields, the distributions of each of the four techniques cannot be expressed in an analytically closed form. This results from not being able to express the distribution of the conventional wavenumber spectra except for infinite velocity signal models. At infinite velocity the distribution of a conventional wavenumber spectra is the uniform sum of chi-square variables. At other velocities the distribution is a non-uniformly weighted sum of chi-square variables whose distribution is not known in closed form. Various approximations to these distributions do exist and can be used if sufficient experimental data is available to define the approximation parameters. This study will provide data from which appropriate approximation can be determined. Finding this approximation is not a major objective of the study, but it may be very useful in analyzing the relative merits of the location scheme being studied.

The statistics are obtained by synthesizing 100 independent transform vectors consisting of 100 noise transform vectors added to a plane wave signal transform vector at the desired signal-to-noise ratio. For the large array the noise vector was made up of zero mean, unit variance random numbers generated by a digital computer. This simulated an uncorrelated noise field. The data from this experiment is being evaluated to determine the locatability of events sensed by a large array.

Further distribution studies will be conducted using array geometries of the extended E3 subarray and a standard LASA subarray to determine the relative performance of the various location schemes in partially correlated noise fields. The results of this study should indicate the relative merits of each approach when operating on data sensed by small arrays.

F. CONCLUSIONS

In view of the above facts, it appears unlikely that the high resolution wavenumber spectra technique will offer any improvement over the conventional beam forming in processing large seismic arrays such as LASA. It is quite possible that computational savings are available by beam-forming in the frequency-domain rather than the time-domain. These computational savings would be a result of the numerical methods utilized in implementing the present techniques rather than the advent of new processing techniques.

Degradation of beam forming performance due to travel-time anomalies and signal waveform distortion across LASA can be quite severe, especially when frequency-domain processing is being considered. Smaller arrays such as the extended LASA subarray (E3) and a standard LASA subarray exhibit smaller travel-time anomalies and waveform distortion than does the large array; but the seismic noise field is correlated between individual elements (seismometers) of the array. Observations of the subarray seismic noise fields (short period) indicate that they are non-isotopic and non-stationary. This situation might require an adaptive scheme to track the noise field in order to achieve the maximum available performance from small array processing.

We believe that it would be fruitful at this time to direct our effort toward comparing the locating ability of an optimum small array probabilistic processor with the location ability of a large array (LASA) using the beam-forming technique and with high resolution f-$\vec{k}$ spectra of small arrays. These comparisons will be conducted using moderate and weak teleseismic events recorded at the Montana LASA when the E3 extended subarray was in operation. This will permit the comparison of the small subarray, the extended subarray, and the large array using the same event for each.

The application of probabilistic processing to arrays with correlated noise fields will require some modification to the present programs, and will not be as numerically efficient as a processor for uncorrelated noise fields. The increased processor complexity might be offset by the capability to accurately locate events using array of small aperture, and the resultant savings in array construction. Determination of the relative capability of large and small arrays when processed in an optimum fashion in the frequency domain is then a reasonable and useful objective of the present study.

G. SUBARRAY PROCESSING

To prepare the raw seismometer data for the detection and location processing, the events being used are processed on a subarray or individual seismometer basis. For the large array processing, the sum of the seismometers on the 1, 3, 4, 5, and 6 rings of each subarray is formed; and this sum is bandpass filtered using a 19-point digital filter. The bandpass filter has a sharp low cut below 0.65Hz and a moderate cutoff above 1.5 Hz and is designed mainly to reduce "leakage" in the Fourier transforms due to the microseismic noise peak at 0.3 Hz. This filter will also be applied to the individual seismometer outputs before the extended E3 and standard LASA subarrays are processed.

The particular choice of seismometers to be summed in the subarray partial sum resulted from studying the array response of several subarray sums as well as the response of an MCF designed to pass greater than 10 km/sec signals and reject 4 km/sec noise. The MCF did not appear to significantly outperform the chosen partial sum to warrant its use. Details of this study will be contained in Large Array Signal and Noise Analysis Special Scientific Report No. 18, SUBARRAY PREPROCESSING FOR DETECTION AND LOCATION.

Very truly yours,

TEXAS INSTRUMENTS INCORPORATED

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Program Manager

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